Stability of Nonexpansive Monotone Systems and Application to Recurrent Neural Networks

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Outline

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Definition: Subtopical systems

A dynamical system with state $x(t) \in \mathbb{R}^n$ at time t, and solution $\varphi(t, x_0) = x(t)$ from the initial condition $x(0) = x_0$, is said to be *topical* if it is:

- **Monotone**:, i.e., solutions preserve the ordering between the initial conditions:

$$y_0 \leq z_0 \Rightarrow \varphi(t, y_0) \leq \varphi(t, z_0), \quad \forall y_0, z_0 \in \mathbb{R}^n, t \geq 0;$$

- **1-(sub)homogeneous**, i.e., solutions are (sub)invariant to rigid transformations:

$$\varphi(t, x_0 + \alpha \mathbf{1}) \leq \varphi(t, x_0) + \alpha \mathbf{1}, \quad \forall x_0 \in \mathbb{R}^n, \alpha \geq 0, t \geq 0.$$

Known fact: trajectories of these systems always converge to equilibrium points (if any exist).

D. Deplano, M. Franceschelli, and A. Giua, "*Novel Stability Conditions for Nonlinear Monotone Systems and Consensus in Multi-Agent Networks*", in IEEE Transaction on Automatic Control (2023)

Main question: does the result hold for general η-subhomogeneity with $\eta \in \mathbb{R}^n_+$?

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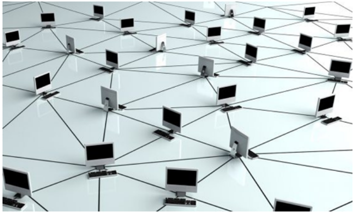
Main question: does the result hold for general $\boldsymbol{\eta}$-subhomogeneity with $\boldsymbol{\eta} \in \mathbb{R}^n_+$?

Multi-Robot Systems



Peer-to-Peer Networks

Chemical Reaction Networks

Neural Networks

- H.L. Smith, "*Monotone Dynamical Systems: An Introduction to the Theory of Competitive and Cooperative Systems*", in American Mathematical Society, (2008).
- D: Angeli and E. Sontag, "*Translation-invariant monotone systems, and a global convergence result for enzymatic futile cycles*", in Nonlinear Analysis: Real World Applications (2008).
- Lemmens and Nussbaum, "*Nonlinear Perron–Frobenius Theory*", in Cambridge Tracts in Mathematics (2012).
- E. Lovisari, G. Como, and K. Savla, "*Stability of monotone dynamical flow networks*", in 53th IEEE Conference on Decision and Control (2014).
- F. Forni and R. Sepulchre, "*Differentially positive systems*", IEEE Transactions on Automatic Control (2015).
- S. Manfredi and D. Angeli, "*Necessary and sufficient conditions for consensus in nonlinear monotone networks with unilateral interactions*", Automatica (2017).
- I.R. Manchester and J.J.E. Slotine, "*On existence of separable contraction metrics for monotone nonlinear systems*", IFAC-PapersOnLine, (2017).
- A. Sootla and A. Mauroy, "*Operator-theoretic characterization of eventually monotone systems*", IEEE Control Systems Letters (2018).
- S. Coogan, "*A contractive approach to separable Lyapunov functions for monotone systems*", Automatica (2019).
- D. Deplano, M. Franceschelli, and A. Giua, "*A nonlinear Perron–Frobenius approach for stability and consensus of discrete-time multi-agent systems*", in Automatica, (2020).
- S. Jafarpour, A. Davydov, and F. Bullo, "*Non-euclidean contraction theory for monotone and positive systems*" IEEE Transactions on Automatic Control (2022).
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A smooth monotone system $\dot{\boldsymbol{x}}(t) = f(\boldsymbol{x}(t))$ is $\boldsymbol{\eta}$-subhomogeneous with $\boldsymbol{\eta} \in \mathbb{R}^n_+$ if and only if it is nonexpansive w.r.t. the diagonally weighted supremum norm

$$\|\boldsymbol{x}\|_{\infty,[\boldsymbol{\eta}]^{-1}} = \max_{i=1,\cdots,n} \frac{1}{\eta_i}|x_i|.$$

Sketch of the proof:

- Change of variable $z(t) = [\eta]^{-1}x(t)$ where $[\eta]$ denotes a diagonal matrix with the entries of η;
- The new system $z(t)$ is monotone and 1-subhomogeneous;
- Under monotonicity, 1-subhomogeneous is equivalent to nonexpansiveness w.r.t. $\|\cdot\|_\infty$ [R1];
- One-to-one relation between trajectories $x(t)$ and $z(t)$;
- The system $x(t)$ is nonexpansive w.r.t. $\|\cdot\|_{\infty,[\eta]^{-1}}$ iff the system $z(t)$ is nonexpansive w.r.t. $\|\cdot\|_\infty$.

[R1] B. Lemmens and R. Nussbaum, Nonlinear Perron-Frobenius Theory. Cambridge University Press, 2012.

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Result 2

Consider a smooth system $\dot{x}(t) = f(x(t))$ satisfying the following:

- the system is monotone and nonexpansive w.r.t. $\|\cdot\|_{\infty,[\eta]^{-1}}$;
- the set of equilibrium points $\mathcal{F}(f) \neq \varnothing$ is not empty.

Then all equilibrium points are stable and each trajectory converges asymptotically to one of them.

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[R2] D. Deplano, M. Franceschelli, and A. Giua, "*Novel stability conditions for nonlinear monotone systems and consensus in multi-agent networks*", IEEE Transactions on Automatic Control (2023).

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How to practically use these results?

Result 3

For a smooth monotone system $\dot{x}(t) = f(x(t))$, the following statements are equivalent:

(a) The system is nonexpansive w.r.t. $\|\cdot\|_{\infty,[\eta]^{-1}}$;

(b) the system is η-subhomogeneous;

(c) the vector field satisfies $f(x + \alpha\eta) \leq f(x)$, $\forall x \in \mathbb{R}^n, \alpha \geq 0$;

(d) the Jacobian satisfies $Df(x)\eta \leq \mathbf{0}$, $\forall x \in \mathbb{R}^n$.

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A simple example

Consider the class of dynamical systems $\dot{\boldsymbol{x}}(t) = f(x(t))$ on $\mathbb{R}^2$ with dynamics:

$$\dot{\boldsymbol{x}}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -x_1(t) + \alpha x_2(t) - \gamma g(x_1) \\ \beta x_1(t) - x_2(t) \end{bmatrix}$$

where $\alpha, \beta, \gamma \geq 0$ and $g : \mathbb{R} \mapsto \mathbb{R}_{\geq 0}$ is C^1 such that $g(0) = 0$ and $g'(x) \geq 0$ for all $x \in \mathbb{R}$.

- Monotonicity holds because the Jacobian $Df(x(t))$ of the vector field is Metzler, indeed,

$$Df(x_1, x_2) = \begin{bmatrix} -1 - \gamma \frac{d}{dt} g(x_1) & \alpha \\ \beta & -1 \end{bmatrix}.$$

- η-subhomogeneity holds for $\alpha\beta \leq 1$, indeed,

$$\begin{cases} -(1 + \gamma \frac{d}{dt} g(x_1(t)))\eta_1 + \alpha \eta_2 \leq 0 \\ \beta \eta_1 - \eta_2 \leq 0 \end{cases} \quad \Rightarrow \quad \eta_2 \in [\beta \eta_1, \frac{1}{\alpha}\eta_1].$$

NB: For $\alpha = 0.5$ and $\beta = 2$ the system is nonexpansive but "non-contracting". Yet, the system converges to one of the equilibrium points.

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We consider two models of RNNs [R3], the Hopfield and the firing-rate models, with dynamics

$$\dot{\boldsymbol{x}}(t) = f_{\mathrm{H}}(\boldsymbol{x}(t)) := -C\boldsymbol{x}(t) + A\Phi(\boldsymbol{x}(t)) + \boldsymbol{b}, \tag{1}$$

$$\dot{\boldsymbol{x}}(t) = f_{\mathrm{FR}}(\boldsymbol{x}(t)) := -C\boldsymbol{x}(t) + \Phi(A\boldsymbol{x}(t) + \boldsymbol{b}), \tag{2}$$

where $C \in \mathbb{R}^{n\times n}$ is a positive diagonal matrix, $A \in \mathbb{R}^{n\times n}$ is an arbitrary matrix, $\boldsymbol{b} \in \mathbb{R}^n$ is a constant input, and $\Phi : \mathbb{R}^n \mapsto \mathbb{R}^n$ is an activation function satisfying the followign assumption.

Assumption 1

Activation functions are diagonal, i.e., $\Phi(\boldsymbol{x}) = [\phi_1(x_1), \cdots, \phi_n(x_n)]^\top$ where each $\phi_i : \mathbb{R} \mapsto \mathbb{R}$ is continuously differentiable and globally Lipschitz, i.e., there exists finite $d_1 \leq d_2$ such that for all $i = 1, \ldots, n$ it holds

$$\frac{d}{dx}\phi_i(x) \in [d_1, d_2], \qquad \forall x \in \mathbb{R},$$

and the Lipschitz constant is given by $\overline{d} = \max\{|d_1|, |d_2|\}$.

[R3] A. Davydov, A.V. Proskurnikov, F. Bullo, "*Non-Euclidean contraction analysis of continuous-time neural networks*", IEEE Transactions on Automatic Control, 2024.

Result 4

Consider Hopfield and firing-rate neural networks as in eqs. (1)-(2) with activation function satisfying Assumption 1. Let $A_\star = \min\{d_1 A, d_2 A\}$ and $A^\star = \max\{d_1 A, d_2 A\}$ satisfy the following conditions:

- $A_\star$ is Metzler (monotonicity);
- $\exists \boldsymbol{\eta} \in \mathbb{R}^n_+ : (A^\star - C)\boldsymbol{\eta} \leq \mathbf{0}$ ($\boldsymbol{\eta}$-subhomogeneity).

Then, all their trajectories converge to some equilibrium point, if any exists.

Some examples of nonexpansive RNNs that are nonexpansive but not contracting can be found for any nonnegative matrix $A \geq 0$ and choosing:

1. $C = \lambda_{\text{MAX}} I$, where λ_{MAX} is the largest eigenvalue of A. In this case, the system is nonexpansive w.r.t. $\|\cdot\|_{\infty, \boldsymbol{v}^{-1}}$ where $\boldsymbol{v}$ is the eigenvector associated with λ_{MAX};
2. $C = \text{diag}(A\mathbb{1})$. In this case, the system is nonexpansive w.r.t. $\|\cdot\|_\infty$;
3. $C = \text{diag}((A\boldsymbol{\eta}))[\boldsymbol{\eta}]^{-1}$ for any $\boldsymbol{\eta} \geq 0$. In this case, the system is nonexpansive w.r.t. $\|\cdot\|_{\infty, \boldsymbol{\eta}^{-1}}$.

NB: This result holds for a class of RRNs that is not considered in the state-of-the-art.

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Contribution 1

- Smooth monotone systems are subhomogeneous if and only if they are nonexpansive w.r.t. a diagonally weighted sup-norm;
- Trajectories of smooth monotone systems that are nonexpansive w.r.t. a diagonally weighted sup-norm converge toward equilibrium points, if any exist;
- Necessary and sufficient conditions for subhomogeneity (and, in turn, nonexpansiveness) are given in terms of the Jacobian matrix of their vector field.

Contribution 2

Application to Recurrent Neural Networks (RNNs) with Hopfield and firing-rate dynamics:

- Monotonicity and subhomogeneity of these neural networks ensure convergence of their state trajectories even if their dynamics are not contractive.

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Application to Recurrent Neural Networks (RNNs) with Hopfield and firing-rate dynamics:

- Monotonicity and subhomogeneity of these neural networks ensure convergence of their state trajectories even if their dynamics are not contractive.

Open questions and future directions

- Do these results hold also for Lipschitz dynamical systems (not necessarily continuously differentiable)?
- How nonexpansiveness is related to subhomogeneity when monotonicity does not hold?
- What is the relation with monotone operators in functional analysis?

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Thank you for your attention!

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